the radar sites to the launch site (about $\frac{1}{2}$ mile). The theoretical trajectory was determined by extrapolation from a series of computed trajectories having sea-level launch angles ranging from 76° to 82° in 2° intervals to obtain the 83° launch-angle equivalent. The theoretical trajectories were based on the equations of motion applying zero-lift, no-wind conditions for a spherical, nonrotating earth (i.e., weight, thrust, and drag). A program applying an n-stage two-dimensional point mass method was used on an IBM 7094 computer. The following conditions applied.

Phase 1: *Nike boost (from sea level), 3.5 sec*
Gross launch weight, 1611.5 lb
Constant mass flow rate and thrust
Drag during burning (see Fig. 2)
Separation at end of Nike burning

Phase 2: *Apache coasting, 16.5 sec*
Drag during coast phase (see Fig. 3)

Phase 3: *Apache burning, 6.4 sec*
Weight at second-stage ignition, 293.5 lb
Variable thrust, about 5000 lb
Drag during burning (see Fig. 3)

Phase 4: *Apache coasting*
Drag, same as phase 2
Atmospheric limit, 400,000 ft ARDC 1959

The results are shown in Fig. 4. It can be seen that the actual and theoretical trajectories disagree slightly. The peak altitude of the theoretical trajectory is higher than the actual by about $3\frac{1}{2}$ miles in about 100 miles. The results of other investigators who have computed theoretical trajectories for the Nike-Apache sounding rocket and their predicted altitudes are also shown in Fig. 4. The results from Ref. 1 would have predicted about 9 miles higher than the actual peak altitude. The results estimated from Ref. 2 (when adjusted for the additional drag of turnstile antennas) would be about 20 miles higher than the actual peak altitude.

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[1] Jenkins, R., *Nike-Apache Performance Handbook*, NASA Goddard Space Flight Center X616-62-103 (July 1962).

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Technical Comments

Hydrodynamic Impact of Conical-Nosed Vehicles during Vertical Water Entry

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It is stated in Ref. 1 of the present comment that "because of their short durations, the impact loadings experienced by a vehicle during water entry will have little effect on the sub-surface trajectory." Upon assuming also that during the downward plunge the vehicle will be balanced on its nose inside the cavity and will continue along a vertical path, the authors present an equation of motion for the vehicle for zero cavitation number as follows:

$$W - \rho_W A h - \frac{\rho_W C_D A}{2g}\left(\frac{dh}{dt}\right)^2 = \frac{W}{g}\frac{d^2h}{dt^2} \qquad (1)$$

where W, ρ_W, g, C_D, A, h, and t are, respectively, the weight, fluid density, acceleration of gravity, forebody drag coefficient, instantaneous cross-sectional area at the surface, depth of penetration below surface, and the time to penetrate a depth h. Then, upon integration of Eq. (1), they are able to arrive at a conservative estimate of the maximum depth to which the vehicle will travel.

If, however, one is interested in calculating the actual maximum force acting on the vehicle and its variation with time, one should not use the solution of Eq. (1). This solution will yield unconservative results, since the effect of impact has been neglected in the derivation of the equation of motion. This does not affect the total depth of penetration considerably, however, since only at instants in the neighborhood of $t = 0$ does the component of force due to impact dominate. However, since here it is desired to calculate the maximum force and its time history, this effect must be included.

This impact effect manifests itself in a so-called "virtual" mass of liquid which must be present as a consequence of the conservation of momentum. Reformulating Eq. (1) to include this effect leads to the following equation:

$$W - \rho_W A h - \frac{\rho_W C_D A}{2g}\left(\frac{dh}{dt}\right)^2 = \frac{d}{dt}(mv) = \frac{d}{dt}\left(m\frac{dh}{dt}\right) \qquad (2)$$

where $m = (M_0 + M)$, and M is the induced virtual mass and M_0 is the original mass of the vehicle.

In the paper by Shiffman and Spencer,[2] an expression for the virtual mass M of the fluid is derived to be

$$M = k(\rho_W/g)h^3 \tan^3\beta \qquad (3)$$

where k is the dimensionless "virtual mass" of the fluid and is a function of the semi-vertex angle β of the cone (see graph 1, Ref. 2). This is done by taking as a first approximation the

Table 1 Maximum force (number of g's)

1 Semi-vertex cone angle, deg	2 Stoff-macher	3 Shiffman & Spencer	4 Watanabe (experi-ments)	5 *A*	6 *B*
70	389	384	402	160	35.8
80	824	838	828	257	33.4
82.5	1105	1138	1145	330	32.7
85	1710	1735	1720	409	32.1

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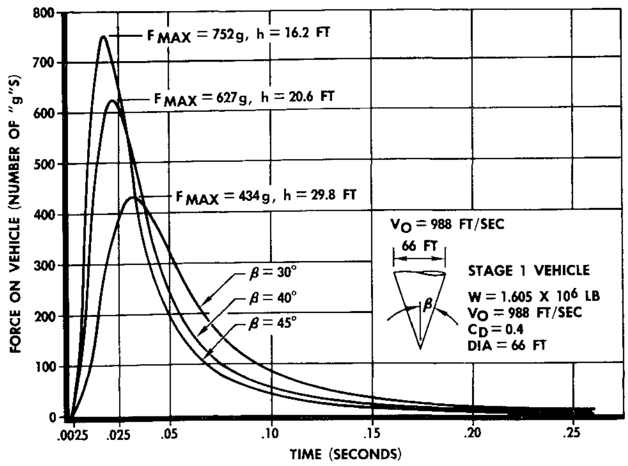

Fig. 1 Hydrodynamic force on vehicle as a function of time.

velocity potential arising from an ellipsoid circumscribed about the cone and then making wetting and free-surface corrections.

The right-hand side of Eq. (2) can be interpreted thusly. The time rate of change of momentum

$$\frac{d}{dt}\left(m\frac{dh}{dt}\right) = \frac{d}{dt}\left[(M_0 + M)\frac{dh}{dt}\right] = (M_0 + M)\frac{d^2h}{dt^2} + \frac{dM}{dt}\frac{dh}{dt} \tag{4}$$

However

$$\frac{dM}{dt} = \frac{dM}{dh}\cdot\frac{dh}{dt}$$

and knowing M as defined by Eq. (3), we get

$$\frac{dM}{dt} = 3k\frac{\rho_W}{g}h^2\tan^3\beta\frac{dh}{dt} \tag{5}$$

Upon substituting Eqs. (3) and (5) into (4) and hence into (2), and noting that the instantaneous cross-sectional area of a vehicle with a conical nose is $A = \pi h^2 \tan^2\beta$, the result is the following equation of motion:

$$A - Bh^3 - Ch^2\left(\frac{dh}{dt}\right)^2 = \frac{d^2h}{dt^2}(D + Eh^3) \tag{6}$$

where $A = W$, $B = \rho_W\pi\tan^2\beta$, $C' = \rho_W C_D \pi \tan^2\beta/2g$, $D = W/g$, $E = k(\rho_W/g)\tan^3\beta$, and $C = C' + 3E$. This is a second-order nonlinear differential equation with constant coefficients, the initial conditions being that at $t = 0$, $h = 0$ and $dh/dt = V_0$ = entry velocity.

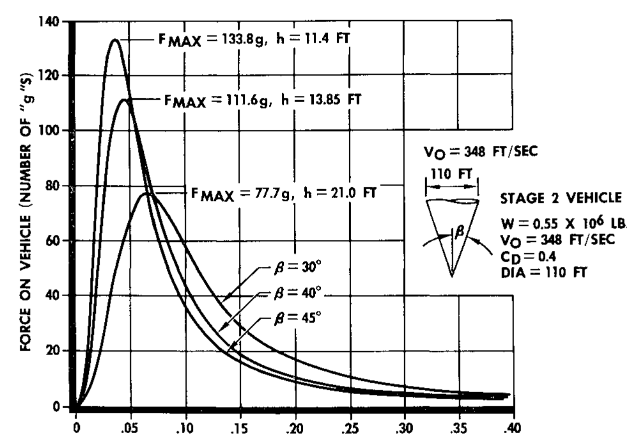

Fig. 2 Hydrodynamic force on vehicle as a function of time.

Equation (6) has been programmed for solution on the IBM 7090 computer using a digital integration program titled "Didas" (for Digital Differential Analyzer). Reference 3 includes the derivation of Eq. (6), its solution by the forementioned program, solution by a Taylor Series approximation, and calculations and graphs of force-time histories for various configurations of recoverable boosters.

Figures 1 and 2 describe the force-time history obtained using the "Didas" solution for two configurations which were studied at Douglas. Noted on the figures are the depths of penetration at which the maximum forces occur.

A comparison of the maximum force determined from this force-time analysis for the stage 2 vehicle of Fig. 2 with values predicted from other analyses is shown in Table 1.

The results in column 4 were calculated from values of the dimensionless "virtual" mass of the fluid k which Watanabe[4] calculated from his experiments. Shiffman and Spencer's values were calculated using Eq. 2.3.10 of Ref. 2.

Column 5, denoted by A, represents the values which are obtained if the impact effect is neglected as in Ref. 1. This can be done by setting $E = 0$ and $C = C'$ in Eq. (6) of this note.

Column 6, denoted by B, represents the value of the force that is calculated from an analysis which makes the assumption, namely, that the virtual mass is picked up by the vehicle with the same instantaneous speed of the vehicle. This assumption is contrary to the one inherent in the calculation of the forces shown in columns 2 and 3 which is that the virtual mass is picked up by the impacting vehicle from a state of rest.

In conclusion it should be emphasized (as was mentioned by the authors of Ref. 1) that unless the geometric configuration is such that it provides a low ballistic parameter, the water entry velocity will be so great as to produce a g level that would be prohibitive even for the most rugged structures.

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[4] Watanabe, S., "Resistance of impact on water surface, Part I," Inst. Phys. and Chem. Res., Tokyo (Scientific Papers) **12**, 251–267 (1930); "Resistance of impact on water surface, Part II," Inst. Phys. Chem. Res., Tokyo (Scientific Papers) **14**, 168–195 (1930).

Landing Loads for Ocean-Recovered Rocket Boosters: Reply by Author to G. Stoffmacher

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In his note, "Hydrodynamic impact of conical-nosed vehicles during vertical water entry,"[1] Stoffmacher claims that the authors of Ref. 2 have neglected the virtual mass of liquid in-

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